

Number of virtual PMUs per unit.	2 Each virtual PMU has its own ID number and configuration. Each PMU may be disabled if not used.
Nominal centre frequency, <b>F</b> Adaptive tuning Bandwidth	50 or 60 Hz Selectable, off or $\pm 2$ , $\pm 5$ , or $\pm 10$ Hz tracking limit See discussion under <i>Window Functions</i>
Phasor estimate reporting rate, <b>R</b> <sup>1</sup>	At 50 Hz: 1, 2, 5, 10, 25, or 50/second At 60 Hz: 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, or 60/second Phasor estimates are batch processed, 20 times per second
Window length, <b>W</b> <sup>1</sup>  Group delay	At 50 Hz: 1 to 16 nominal cycles (integer values) At 60 Hz: 1 to 16 nominal cycles W/2F, in seconds
Estimator algorithm	Selectable: Raised cosine per de la O and Martin, 2003 [1]; Hann; Hamming; Blackman; Triangular (Bartlett); Rectangular; Flat Top; Kaiser; Nuttall 4-term [3] (see section on <i>Window Functions</i> ).
Measurement accuracy	0.1% Total Vector Error (TVE) maximum <sup>2</sup> , plus estimator error (generally small with proper configuration; can be modelled for suitability for a particular application; see <i>Selecting Windows and Their Parameters</i> )
Input signals processed	A, B, and C channel voltage A, B, and C channel current (both selectable on/off) <sup>1,3</sup>
Phasor estimate reporting formats	A, B, and C per-phase data Positive-sequence component Negative and zero-sequence components (each group selectable on/off) <sup>4</sup> Fixed or floating-point <sup>4</sup>
Data formatting compatibility	Per IEEE Standard C37.118-2005 [2], serial or Ethernet

**Table 1: 1133A Phasor Measurement Specifications**

<sup>1</sup>There is a limit on the total number of cycles calculated as follows: the value  $(R1 \cdot W1) + (R2 \cdot W2)$  must not exceed 400. R1 and R2 are the phasor estimate reporting rates for virtual PMU #1 and #2, respectively and W1 and W2 are the respective window lengths in cycles. See example calculation under 'Virtual PMU,' next page. The maximum reporting rate of either 50 or 60 per second is only available on one PMU. If 50 or 60 per second is selected for a PMU the other PMU must be configured for a lower rate.

<sup>2</sup>All corrections for internal offsets and system configuration (for example, PT and CT ratios and their errors) are performed automatically and are included in the 0.1% limit. This limit presumes operation under specified conditions, including lock to a suitable time base such as GPS, and includes all errors resulting from such operation. See the Model 1133A Technical Data for more information, including performance degradation caused by operation without a reference time base.

<sup>3</sup>Voltages and currents may be selected independently for each virtual PMU. Voltages and currents are selected as a three-phase entity, i.e. selection of 'voltage' enables calculation for all three voltage inputs A, B and C. The per-phase inputs are not individually selectable.

<sup>4</sup>Each of the three selectable blocks of data (per-phase, positive sequence, and negative/zero sequence) and fixed/floating point format may be independently selected for each virtual PMU. The separate phases and negative/zero sequence are not individually selectable. The selection of fixed- or floating-point applies to all selections for a given virtual PMU.

## Model 1133A Background

The Model 1133A Power Sentinel includes a flexible, floating-point digital signal processor (DSP). This powerful DSP has allowed Arbiter Systems to significantly enhance functionality of the Model 1133A since its introduction. **These enhancements are free to all users of the Model 1133A**, requiring only a firmware download to enable the most recent features. (Some early units require the installation of a new DSP boot ROM to enable firmware download. Contact the factory for a free boot ROM if required.)

The initial implementation of phasor measurements in the Model 1133A was based on its 20/second common measurement rate, used for all basic internal functions (voltage, current, power, frequency, etc.) This 20/second data, adequate for many applications, was simply output in phasor format. With the enhancements described here, variable update rates from 1 to 240/second are supported, with individually-selectable configuration parameters for each of two virtual phasor measurement units (PMUs).

**Latency:** Although phasor estimates are now available at reporting rates of up to 60/second, the actual calculation and formatting of the results is still performed once every 50 ms, as each new input data block becomes available.

Therefore, latency estimates must include an allowance for this batch processing. Latency for each block can vary from  $(W/2F)+30$  ms for the most recent phasor estimate to  $(W/2F)+80$  ms for the oldest estimate within each batch. W and F are defined in the specification table above. W/2F is the inherent group delay of a symmetric FIR estimator. Keep in mind that latency estimates are approximate, and can be significantly longer depending on the communications channel. The numbers here are for an Ethernet network without significant added delay due to collisions. **Latency** is defined here as the time between the phasor time tag (at the middle of each window) and the time when data transmission begins.

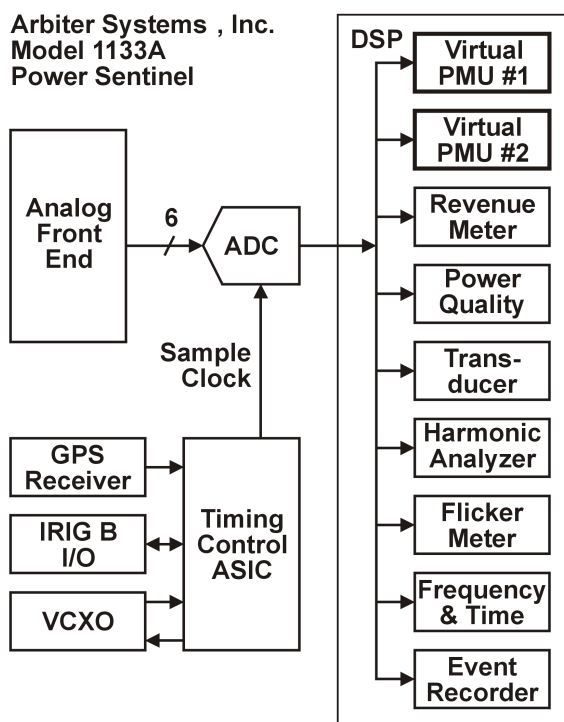


Figure 1: Model 1133A simplified block diagram

## Virtual PMU

The virtual PMU is implemented as a separate processing block in the DSP, based on a common set of sampled input data from the unit's three-phase voltage and current input section (figure 1). The hardware is shared between the two virtual PMUs, but each has its own set of configuration parameters, all of which, including the PMU ID field, can be selected completely independently of the other virtual PMU. It is even possible to set one PMU for 50 Hz operation and one for 60 Hz operation, although it is unlikely that such a combination would be very useful. However, in some applications, it is useful to have phasor data at different rates, or in different formats, or using estimators optimised with different characteristics.

For instance, a Model 1133A could be configured with two virtual PMUs as follows. One PMU might measure data at a once-per-cycle rate (50 or 60 phasors per second), giving floating-point results for all six inputs (three voltage + three current), with a 4-cycle measurement window. This PMU could deliver its results via Ethernet for local control applications. A second PMU, optimised for wide-area monitoring (WAM), might provide phasor estimates at a lower rate, say 10/second, of positive-sequence voltage only, formatted in fixed point for transmission over a low-bandwidth channel to a remote phasor data concentrator. This PMU could use a longer window, 10 or 12 cycles long, with a correspondingly narrower bandwidth to provide anti-aliasing. Adaptive centre-frequency tuning could be used in this PMU, to eliminate rolloff (magnitude errors) for off-nominal frequency signals, even with the narrower bandwidth.

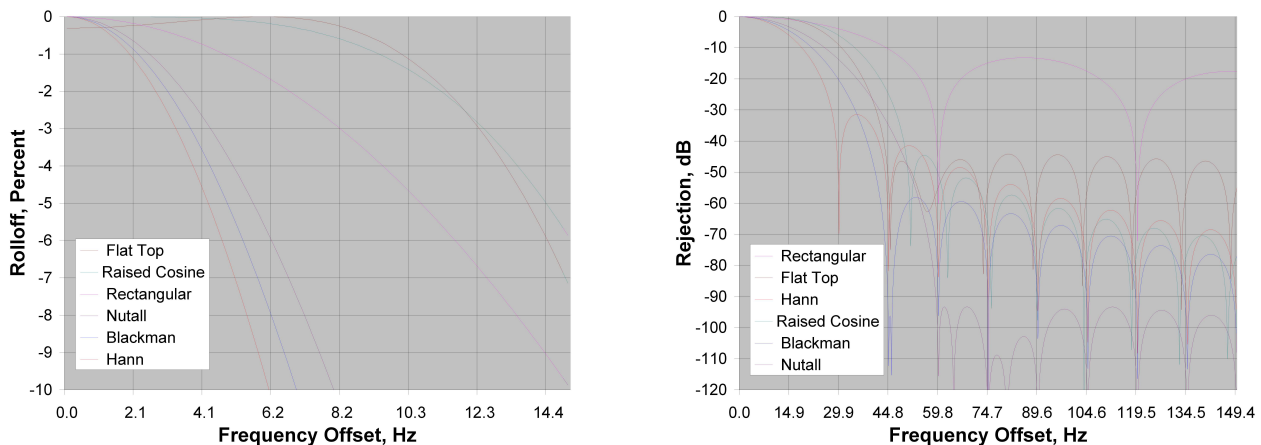
For this example, the total number of calculated cycles is as follows (using 60 Hz for the example):  $R1 = 60/\text{second}$ ;  $W1 = 4$  cycles:  $R1 \cdot W1 = 240$ .  $R2 = 10/\text{second}$ ;  $W2 = 12$  cycles:  $R2 \cdot W2 = 120$ .  $240 + 120 = 360$ , under the limit of 400. Similar results can be calculated for a 50 Hz system. So, the choice of parameters, particularly reporting rates, is not

usually limited by hardware capabilities, but can be optimised for the application. For example, you could change the configuration of virtual PMU #2, used for the WAM function, to provide data at a higher rate using a shorter window to provide a wider measurement bandwidth when the wide-area network is updated, without affecting the configuration for virtual PMU #1.

## Window Functions

The Model 1133A offers a wide range of window functions. The choice can be optimised for each application. All window functions serve the same purpose (as a low-pass filter) and work in the same basic way. The main difference between windows is the shape and magnitude of ‘sideband lobes,’ which are peaks in the rejection band. The various window functions also have differences in passband width and flatness. Figures 2 and 3 show the in-band and rejection responses of several of the available windows. All of these windows are shown for  $F=60$ ,  $W=4$  except the rectangular window, which is shown for  $W=1$  for comparison with first-generation PMUs, which used this function. For the raised cosine window,  $K=2$  and  $\alpha=0.7$ . Curves for 50 Hz are similar, multiplying the frequency scale by 5/6.

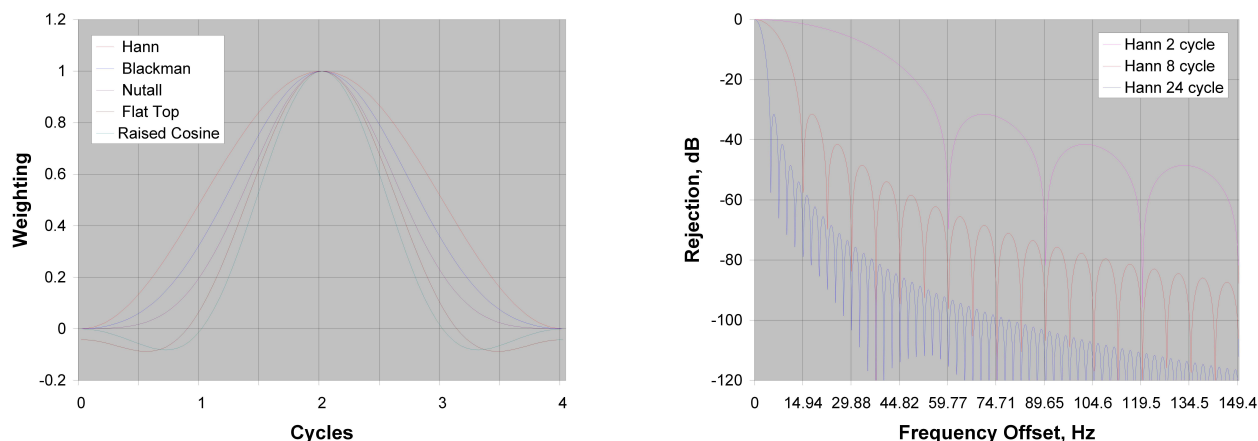
Window functions are also called ‘weighting functions,’ because they work by multiplying the input signal time record by an equal-length sequence of constants, or weighting factors. Most window functions decrease smoothly to zero at their ends (see figure 4). The rectangular window is an exception: all of its values are 1.0, and it is equivalent to no window at all. It has the narrowest main lobe (passband) of any window (for a given width), and works well only when the signal is centred in the passband, i.e. at nominal system frequency. Its performance is worse than any other window for off-nominal and out-of-band signals. The differences in rolloff between the various window functions become more apparent as the signal moves further from nominal (greater frequency offset).



**Figures 2 and 3: Performance of selected window functions**

Note that all windows in the Model 1133A offer variable width in terms of nominal power-system cycles, with window lengths up to 320 ms available. The shortest practical window length varies and is generally longer for higher-performance windows. This is a general characteristic of finite-impulse-response (FIR) filters, of which windowed estimators are one example: performance improves using longer windows, or equivalently, more data samples. ‘Performance’ here means improved in-band flatness, narrower transition band, and/or increased out-of-band rejection. However, as performance measured by these parameters improves, group delay increases.

For all window functions, as window length increases the passband narrows in inverse proportion (figure 5). Sideband lobes also narrow and move closer to the passband, while their magnitude stays relatively constant. For some of the window functions (raised cosine and Kaiser), other constants also affect the shape and performance of the window. For the raised-cosine window, the 6-dB bandwidth is a function of the constants  $K$  and  $\alpha$ , as well as the window length: passband width is proportional to  $K/W$ .



**Figures 4 and 5: Window shape and effect of window length**

All of the windows used in the Model 1133A are symmetric, which means that they have a constant group delay equal to half the window length,  $W/2F$ . Applications that need fast response must therefore accept tradeoffs in performance measured in the frequency domain. Where greater accuracy or more rejection is needed, and response time is less critical, the Model 1133A easily produces estimators based on longer, higher-performance windows that deliver the needed accuracy.

Note that with adaptive tuning (see below), a broad passband is not required, since the estimator centre frequency follows the applied signal. The window function can be selected for its rejection and group delay characteristics only. Indeed, a narrower passband aids in rejecting noise.

References [3] and [4] provide good background on the derivation and use of window functions.

## Selecting Windows and Their Parameters

The acceptance criteria for a window should include its 1% bandwidth (wider is better) and rejection at  $2(F \pm \Delta F)$ , typically 100-140 Hz for a 60 Hz system, or 80-120 Hz at 50 Hz. To minimise errors due to feedthrough of this double-frequency term, rejection should be -40 dB minimum, and -60 dB is better. If anti-aliasing for low reporting rates is an issue, rejection will be important at much lower frequencies, as well.

For most applications, the rectangular and triangular (Bartlett) windows are not recommended, since the other windows offer superior performance. These two are provided mostly for experimental purposes. The often-used Hann (sometimes called Hanning) and Blackman windows both have the desirable characteristic that the magnitude of their rejection sidelobes decreases with increasing frequency. They also have reasonable passband characteristics, especially when used with adaptive tuning, and where anti-aliasing is an issue. The Hamming window is similar to the Hann, but since the window does not go to zero at the ends, its rejection sidelobes do not decrease as quickly. The first sidelobe is, however, smaller than for the Hann. The Nuttall 4-term window [3] is similar to the Hann and Blackman, with even better rejection characteristics (>90 dB). The Hann and Hamming (2-term), Blackman (3-term) and Nuttall (4-term) windows all use the same general equation (as does the flat-top, discussed later). These are called the *Blackman-Harris family* of windows:

$$w(n) = a_0 - a_1 \cos(2\pi n/N) + a_2 \cos(4\pi n/N) - a_3 \cos(6\pi n/N); 0 \leq n \leq N$$

Window	$a_0$	$a_1$	$a_2$	$a_3$
Hann	0.5	0.5		
Hamming	0.54	0.46		
Blackman	0.42	0.5	0.08	
Nuttall	0.355768	0.487396	0.144232	0.012604
Flat-top	0.2810639	0.5208972	0.1980399	

**Table 2: Blackman-Harris family parameters**

The Kaiser window has performance similar to the Blackman-Harris family, although its calculation is completely different, and based on a Bessel function. Its parameter,  $\beta$ , allows it to emulate the Hann ( $\beta=6$ ) and Blackman ( $\beta=8.6$ ) windows, plus a variety of other similar characteristics.

The flat-top window has a broader passband than the previous windows, which might make it useful in applications where adaptive tuning is not desirable. Its passband is not truly flat ( $-0.32\%$  at  $f=0$ ), and its rejection is not as good as the others, as can be seen in Fig. 3, but it is acceptable under many conditions and far better than the rectangular window. The raised-cosine window [1] offers the broadest flat passband, along with rejection comparable to the Hann window for the conditions shown. For the raised-cosine filter, as  $\alpha$  decreases, giving a sharper cutoff and broader passband,  $K$  and  $W$  must increase. If  $\alpha$  is too small for  $K$  and  $W$ , large inaccuracies in the frequency response will result, due to excessive truncation of the (theoretically infinite) impulse response used to define the window. Since the configuration utility places no restrictions on these parameters, it is the user's responsibility to verify that the chosen parameters give acceptable performance. The parameter  $K$  is equal to the number of 'symbol times' ( $T_s$ ) which fit into half of the window  $W$ . The term  $T_s$  comes from the digital communications field, which uses this family of filters extensively. Note that  $K$  must be an integer so that the window function reaches zero at its end points.

Cutoff frequency (-6 dB bandwidth)	$\pm KF/W$ ; $1 \leq K \leq (W/2)$ ; $K$ an integer
Rolloff factor, $\alpha$	0.0 – 1.0      Example: $F=60$ , $W=4$ , $K=2$ , $\alpha=0.7$ :
Flat passband (nominal)	$\pm(1-\alpha)KF/W$ $\pm 9$ Hz (relative to centre frequency)
-6 dB bandwidth	$\pm KF/W$ $\pm 30$ Hz
Transition bandwidth	$2\alpha KF/W$ 42 Hz
Rejection band (nominal)	$\pm(1+\alpha)KF/W$ $\pm 51$ Hz

**Table 3: Raised-cosine filter characteristics**

To aid in window selection, Arbiter Systems offers a computer program that simulates the performance of all of these filters, with the window length and other parameters as variables. This program allows you to evaluate each weighting function against the requirements of your application, and see the results either in graphic or summary form.

## Adaptive Tuning

A phasor measurement unit is similar in concept to a digital communications receiver (figure 6). The input signal is convolved (multiplied in the time domain) with a complex sine wave at the nominal receiver (power system) frequency. The resulting baseband signal is low-pass filtered to eliminate double-frequency components and interfering signals. The output of this filter is the *analytic representation* of the input signal, with in-phase (I) and quadrature (Q) components that can be transformed into magnitude and phase angle. The low-pass filter in the Model 1133A is produced by the window function, described in the previous section.



As with a communications receiver, when the input signal moves away from the nominal receiver frequency, the resulting signal is attenuated by the filter frequency response. While filters can be designed to minimise attenuation over a certain bandwidth, there is still rolloff outside this band, and these filters may not have optimum characteristics for all applications.

Rolloff effects can be reduced (practically eliminated) by continuously adjusting the receiver centre frequency to track the incoming signal (**adaptive tuning**), thereby keeping the convolver output in the centre of the receiver passband as the input frequency changes. However, this introduces the possibility that the frequency could mis-tune enough to prevent proper operation. For an effective and reliable adaptive-tuning algorithm, we must address this possibility.

In the Model 1133A, the PMU centre frequency (when adaptive tuning is enabled) is set by the frequency measured with the Model 1133A's multi-purpose frequency-transducer function. This measurement is made by determining  $w = \Delta\phi/\Delta t$  of V1, the positive-sequence voltage. This measurement has sufficient bandwidth that the signal will never get 'lost.' Combined with a limit of  $\pm 2, 5$ , or 10 Hz maximum tuning range, errors due to rolloff can be eliminated while ensuring that the algorithm never 'loses' the signal. As an additional 'sanity check,' the PMU magnitude results are cross-checked against the broadband rms results from the 1133A's multi-purpose transducer function, and adaptive tuning is disabled if the error exceeds a preset limit.

Adaptive tuning cannot be perfect, and it is worthwhile to consider the effects of slight mistuning on the PMU response. The effect on magnitude response is quite straightforward. Any errors are due to the very same filter rolloff that started the discussion. However, most filters are quite insensitive to mistuning near their centre frequency with respect to magnitude response. Frequency offsets are random, caused by noise, and are generally much less than 0.01 Hz. Since the smallest 6 dB bandwidth possible with the window functions in the Model 1133A is approximately 5 Hz, it can be shown that frequency noise will have a negligible effect – usually much less than 0.01%. Phase-angle accuracy is even less dependent on frequency offsets, being totally insensitive to the centre frequency. Measured phase angle for any centre frequency and signal frequency combination are identical, provided that the output of the low-pass filter is far enough above the noise floor.

Low-pass rejection, due to the window function, also must be considered since the double-frequency (image) product of the convolution process will alias into the passband (for any  $R \leq 2F$ ), and so must be rejected. Note that this is also true if the frequency is at nominal, but most windows, even the rectangular window, have near-infinite rejection notches in the vicinity of the harmonics of  $F/W$  (e.g., starting with  $N = 2 \dots 4$  for the Blackman-Harris family). Rejection should be at least 40 dB, and 60 dB is better, at  $2F$  *plus or minus expected frequency offsets*. Indeed, this is one of the major limitations of first-generation PMUs. Off-nominal frequency signals aliased into the passband due to the limited rejection of the one-cycle rectangular window (figure 3), causing significant errors [1].

In summary, adaptive tuning allows relatively-narrow estimator bandwidths, to provide anti-aliasing and rejection of out-of-band signals, while still providing excellent accuracy measuring the magnitude and phase of the power-system signal. This is particularly useful for lower phasor reporting rates, below  $R = F/2$  per second.

## Synchronisation and ADC Resolution

Input sampling in the Model 1133A is performed using a time-multiplexed 14-bit A/D converter, synchronised to UTC via the Global Positioning System to within one microsecond anywhere on earth (see figure 1). There has been some discussion about the A/D conversion process as it relates to phasor estimation, particularly with respect to synchronisation and resolution. Opinions have been expressed as fact, without a full appreciation of the nuances involved in the design of sampled data systems. This has led to incorrect or oversimplified generalisations and misunderstandings.

Synchrophasors can be estimated in a number of ways. Contrary to common belief, there is no absolute requirement for synchronised data samples or a particular ADC resolution in order to meet the requirements of the IEEE synchrophasor standards. What **is required** by these standards is that the measurement (phasor estimate) be within

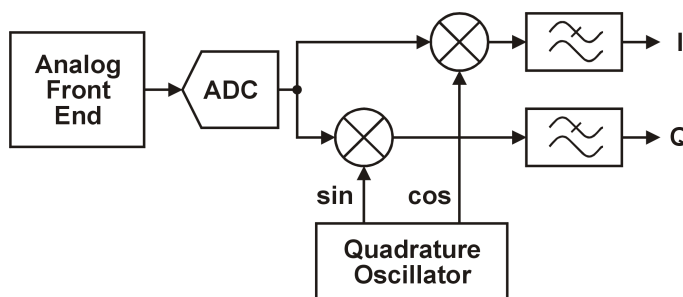


Figure 6: Digital receiver block diagram

specified limits ( $\pm 1\%$  total vector error) at the reporting times specified. Phasors could be measured with a free-running sample clock, provided some means is provided to determine the relationship between the ADC clock and UTC, and to transform the output so as to match the specified reporting times. While this is possible, it is not trivial, and we are not aware of any commercial implementations using this approach. Nevertheless, such an implementation properly done should give results which are equivalent (in the steady state, for which the standards apply, other factors being equal) to implementations using synchronised sampling. Note that this is **not** simply time-tagging the results. The IEEE standards require that the phasor estimates be made *at prescribed points in time*, not simply tagged with the time they were made. This requirement is intended to maximise compatibility between PMUs from different makers.

There is also no requirement for input sampling to be time-aligned with e.g. 1PPS-UTC. The **reported results** must be aligned – but this is an important difference. It is in fact impossible to sample a signal ‘exactly’ at the time of occurrence of 1PPS-UTC, or at any other specific point in time. It is also unnecessary. If the sampling time is offset, then either (a) the amount of offset must be small enough so as not to compromise performance; in other words, the PMU must meet the 1% TVE requirement without correction for the offset; or (b) the PMU must compensate for the sampling offset. The standards contemplate this and allow for it (indeed they require it). Actually, one good method to compensate for analog anti-alias filter delay is to intentionally delay the sampling clock by an equivalent amount.

There is also no requirement for a specific ADC resolution. There is a widespread misconception that performance is limited in a fundamental way by the number of bits in the ADC. While there are several ADC parameters that can place fundamental limitations on performance – linearity, noise and gain stability for example – resolution is not one of them, at least not in itself.

For example, many modern systems (including some PMUs) use ‘sigma-delta’ A/D converters. Engineers often view these in terms of the number of rated output bits, commonly 16 to 24. We do not pay much attention to the fact that these are really **one-bit A/D converters**, integrated with a digital filter which provides the enhanced output resolution.

The keys to understanding this seeming paradox are **processing gain** and **noise shaping**. A one-bit converter has a theoretical signal-to-noise ratio of 1.76 dB for a full-scale signal. This might seem hardly useful for anything. However, if this converter is made to operate at a rate much higher than the measurement bandwidth, then the noise, spread over the entire Nyquist bandwidth of dc to one-half of the sampling frequency, is reduced greatly by filtering since most of it is outside the bandwidth of the filter. For example, a one-bit converter operating at one million samples per second (1 MSPS) and used with a PMU having a 60 Hz measurement bandwidth could deliver a S/N ratio of about 41 dB – adequate (barely) to meet the 1% TVE requirement. The ratio of 60 Hz to 500 kHz (the Nyquist bandwidth), which is 8333 or 39.2 dB, is called **processing gain**. The processing gain is greater for narrower measurement bandwidths, and for higher sampling rates. In general, a converter having fewer bits and a higher sampling rate can provide performance equivalent to a converter with more bits and a lower sampling rate. It depends on the other parameters of the ADC, primarily linearity and noise, and on the use of the proper DSP algorithms. Engineers who design sampled-data systems understand and make these tradeoffs routinely.

The preceding discussion presumes that the sampling noise is distributed evenly across the entire frequency band from dc to the Nyquist limit. (For quantization noise in an ADC, this is true if the noise is not correlated to the input signal. This is a whole different subject, but the noise is mostly non-correlated if the input signal is large relative to ADC resolution, and/or the ratio of sampling to signal frequency is large. Correlated ‘noise’ produces harmonics and intermodulation products.) Consider what would happen, however, if the noise could be ‘shaped’ in the frequency domain so that more of it was outside the desired measurement bandwidth. If we could shape the noise spectrum, moving the quantization noise away from the desired signal, then the processing gain could be greater than simply the ratio of the bandwidths. This can in fact be done, by including the ADC inside a feedback loop having one or more integrators. The integrators reduce the amount of noise at low frequencies, at the expense of increasing it at high frequencies. Note that the *total noise is always the same*, and there is no ‘free lunch’ – all we are doing is moving the noise to a higher frequency, where it is attenuated by subsequent low-pass filtering. This process is called **noise shaping**, and is what makes it possible to achieve signal-to-noise ratios of 100 dB or better with what is basically a one-bit converter.

**Conclusion:** there is no simple relationship between synchronisation method, ADC resolution, and synchrophasor measurement performance. There is a relationship, of course, but not a **simple** one. We believe that the process implemented in the Model 1133A is a good one, with respect to cost/performance tradeoffs, but we recognise that there are many other ways to solve this problem. The manufacturer’s job in building a PMU is to understand the various tradeoffs involved and select an implementation which optimises performance while meeting cost and reliability objectives.

## References

- [1] J. A. de la O Serna and K. E. Martin, "Improving Phasor Measurements Under Power System Oscillations," IEEE Trans. Power Sys., Vol. 18, No. 1, Feb. 2003.
- [2] IEEE Standard C37.118-2005, "Synchrophasors for Power Systems."
- [3] A. H. Nuttall, "Some Windows with Very Good Sidelobe Behavior," IEEE Trans. Acoustics, Speech and Signal Processing, Vol. ASSP-29, No. 1, Feb. 1981.
- [4] F. J. Harris, "On the Use of Windows for Harmonic Analysis with the Discrete Fourier Transform," Proc. IEEE, Vol. 66, No. 1, Jan. 1978.